

Floating Point Routines for the 6502

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and

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Editor's Note: Although these routines are for the 6502, it would appear that one could generate equivalent routines for most of the "traditional" microprocessors, relatively easily, by following the flow of the algorithms given in the excellent comments included in the program listing. This is particularly true of the transcendental functions which were directly modeled after well-known and proven algorithms, and for which, the comments are relatively machine-independent.

These floating point routines allow 6502 users to perform most of the more popular and desired floating point and transcendental functions, namely:

- Natural Log - LOG
- Common Log - LOG10
- Exponential - EXP
- Floating Add - FADD
- Floating Subtract - FSUB
- Floating Multiply - FMUL
- Floating Divide - FDIV
- Convert Floating to Fixed - FIX
- Convert Fixed to Floating - FLOAT

They presume a four-byte floating point operand consisting of a one-byte exponent ranging from -218 through +127, and a 24-bit two's complement mantissa between 1.0 and 2.0.

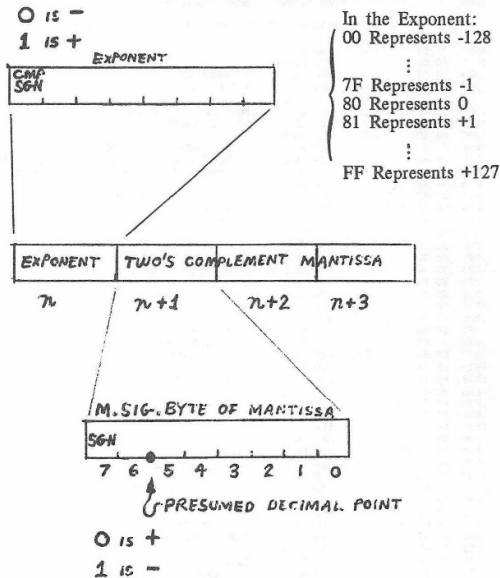
The floating point routines were done by Steve Wozniak, one of the principals in Apple Computer Company. The transcendental functions were patterned after those offered by Hewlett-Packard for their HP2100 minicomputer (with some modifications), and were done by Roy Rankin, a Ph.D. student at Stanford University.

There are three error traps; two for overflow, and one for prohibited logarithm argument. ERROR (1D06) is the error exit used in event of a non-positive log argument. OVFLW (1E3B) is the error exit for overflow occurring during calculation of e to some power. OVFL (1FE4) is the error exit for overflow in all of the floating point routines. There is no trap for underflow; in such cases, the result is set to 0.0.

All routines are called and exited in a uniform manner: The argument(s) are placed in the specified floating point storage locations (for specifics, see documentation preceding each routine in the listing), then a JSR is used to enter the desired routine. Upon normal completion, the called routine is exited via a subroutine return instruction (RTS).

Note: The preceding documentation was written by the Editor, based on phone conversations with Roy and studying the listing. There is a high probability that it is correct. However, since it was not written nor reviewed by the authors of these routines, the preceding documentation may contain errors in concept or in detail.

- JCW, Jr.



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*
* JULY 5, 1976
* BASIC FLOATING POINT ROUTINES
* FOR 6502 MICROPROCESSOR
* BY R. RANKIN AND S. WOZNIAK
*
* CONSISTING OF:
* NATURAL LOG
* COMMON LOG
* EXPONENTIAL (E**X)
* FLOAT      FIX
* FADD      FSUB
* FMUL      FDIV
*
* FLOATING POINT REPRESENTATION (4-BYTES)
* EXPONENT BYTE 1
* MANTISSA BYTES 2-4
*
* MANTISSA: TWO'S COMPLEMENT REPRESENTATION WITH SIGN IN
* MSB OF HIGH-ORDER BYTE. MANTISSA IS NORMALIZED WITH AN
* ASSUMED DECIMAL POINT BETWEEN BITS 5 AND 6 OF THE HIGH-ORDER
* BYTE. THUS THE MANTISSA IS IN THE RANGE 1. TO 2. EXCEPT
* WHEN THE NUMBER IS LESS THAN 2**(-128).
*
* EXPONENT: THE EXPONENT REPRESENTS POWERS OF TWO. THE
* REPRESENTATION IS 2'S COMPLEMENT EXCEPT THAT THE SIGN
* BIT (BIT 7) IS COMPLEMENTED. THIS ALLOWS DIRECT COMPARISON
* OF EXPONENTS FOR SIZE SINCE THEY ARE STORED IN INCREASING
* NUMERICAL SEQUENCE RANGING FROM 000 (-128) TO 0FF (+127)
* ($ MEANS NUMBER IS HEXADECIMAL).
*
* REPRESENTATION OF DECIMAL NUMBERS: THE PRESENT FLOATING
* POINT REPRESENTATION ALLOWS DECIMAL NUMBERS IN THE APPROXIMATE
* RANGE OF 10**(-38) THROUGH 10**(38) WITH 6 TO 7 SIGNIFICANT
* DIGITS.
*
0003          ORG 3          SET BASE PAGE ADDRESSES
0003 EA      SIGN      NOP
0004 EA      X2      NOP
0005 00 00 00 12      BSS 3      MANTISSA 2
0006 EA      X1      NOP
0007 00 00 00 11      BSS 3      MANTISSA 1
0008          E      BSS 4      SCRATCH
0010          Z      BSS 4
0014          T      BSS 4
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0010      SEXP BSS 4
001C      INT BSS 1
*
1000      * ORG $1000 STARTING LOCATION FOR LOG
*
* NATURAL LOG OF MANT/EXP1 WITH RESULT IN MANT/EXP1
*
1000 A5 09 LOG LDA M1
1002 F0 02 BEO ERROR
1004 10 01 BPL CONT IF ARG>0 OK
1006 00 ERROR BRK ERROR ARG<0
*
1007 20 1C IF CONT JSR SWAP MOVE ARG TO EXP/MANT2 < A2.00 LIX# 0
100A A5 04 LDA X2 HOLD EXPONENT
100C A0 00 LDY #000
100E 04 04 STY X2 SET EXPONENT 2 TO 0 (000)
1010 49 00 COR #000 COMPLIMENT SIGN BIT OF ORIGINAL EXPONENT
1012 05 00 STA M1+1 SET EXPONENT INTO MANTISSA 1 FOR FLOAT
1014 A9 00 LDA #0 BDL#*3
1016 05 09 STA M1 CLEAR MSB OF MANTISSA 1 DEX< 86.09 37X A1
1018 20 2C IF JSR FLOAT CONVERT TO FLOATING POINT
101A A2 03 LDX #3 4 BYTE TRANSFERS
101D 05 04 SEXP1 LDA X2.X
101F 95 10 STA Z.X COPY MANTISSA TO Z
1021 05 00 LDA X1.X
1023 95 10 STA SEXP.X SAVE EXPONENT IN SEXP
1025 00 D1 1D LDA R22.X LOAD EXP/MANT1 WITH SORT(2)
1028 95 00 STA X1.X
102A CA DEX
102C 10 F0 BPL SEXP1 Z-SORT(2)
102D 20 4A IF JSR FSUB
102E A2 03 LDX #3 4 BYTE TRANSFER
1030 05 00 LDA X1.X SAVE EXP/MANT1 AS T
1032 95 14 SAVET STA T.X
1034 05 10 LDA Z.X LOAD EXP/MANT1 WITH Z
1036 05 00 STA X1.X
1038 05 00 LDA R22.X LOAD EXP/MANT2 WITH SORT(2)
103A 00 D1 1D STA X2.X
103D 95 04 DEX
103F CA BPL SAVET
1040 10 F0 JSR FADD Z+SORT(2)
1042 20 50 IF LDX #3 4 BYTE TRANSFER
1044 A2 03 LDX #3
1046 05 14 TH2 LDA T.X
1048 95 04 STA X2.X LOAD T INTO EXP/MANT2
104A CA DEX
104C 10 F9 JSR TH2
104E 20 9D IF JSR FDIV T=(Z-SORT(2))/(Z+SORT(2))
1050 A2 03 LDX #3 4 BYTE TRANSFER
1052 05 00 LDA X1.X
1054 95 14 MIT STA T.X COPY EXP/MANT1 TO T AND
1056 95 04 STA X2.X LOAD EXP/MANT2 WITH T
1058 CA DEX
105A 10 F7 BPL MIT
105C 20 77 IF JSR FSUB T=T-T
105E 20 1C IF JSR SWAP MOVE T+T TO EXP/MANT2
1060 A2 03 LDX #3 4 BYTE TRANSFER
1062 00 E1 1D M1C LDA C.X
1064 95 00 STA X1.X LOAD EXP/MANT1 WITH C
1066 CA DEX
1068 10 F0 BPL M1C
106A 20 4A IF JSR FSUB T=T-C
106C A2 03 LDX #3 4 BYTE TRANSFER
106E 00 00 1D M2MB LDA MB.X
1070 95 04 STA X2.X LOAD EXP/MANT2 WITH MB
1072 CA DEX
1074 10 F0 BPL M2MB
1076 20 9D IF JSR FDIV MB/(T+T-C)
1078 A2 03 LDX #3 4 BYTE TRANSFER
107A 00 D9 1D M2A1 LDA A1.X
107C 95 04 STA X2.X LOAD EXP/MANT2 WITH A1
107E CA DEX
1080 10 F0 BPL M2A1
1082 20 50 IF JSR FADD MB/(T+T-C)+A1
1084 A2 03 LDX #3 4 BYTE TRANSFER
1086 05 14 M2T LDA T.X
1088 95 04 STA X2.X LOAD EXP/MANT2 WITH T
108A CA DEX
108C 10 F9 BPL M2T
108E 20 77 IF JSR FSUB (MB/(T+T-C)+A1)+T
1090 A2 03 LDX #3 4 BYTE TRANSFER
1092 00 E5 1D M2ML LDA M1L.F.X
1094 95 04 STA X2.X LOAD EXP/MANT2 WITH M1L.F. (.5)
1096 CA DEX
1098 10 F0 BPL M2ML
109A 20 50 IF JSR FADD +.5
109C A2 03 LDX #3 4 BYTE TRANSFER
109E 05 04 LDEXP LDA SEXP.X
10A0 95 04 STA X2.X LOAD EXP/MANT2 WITH ORIGINAL EXPONENT
10A2 CA DEX
10A4 10 F9 BPL LDEXP
10A6 20 50 IF JSR FADD +EXP
10A8 A2 03 LDX #3 4 BYTE TRANSFER
10AA 00 D5 1D MLE2 LDA LE2.X
10AC 95 04 STA X2.X LOAD EXP/MANT2 WITH LN(2)
10AE CA DEX
10B0 10 F0 BPL MLE2
10B2 20 77 IF JSR FSUB LN(2)
10B4 68 RTS RETURN RESULT IN MANT/EXP1
*
* COMMON LOG OF MANT/EXP1 RESULT IN MANT/EXP1
*
10B8 20 00 1D LOG10 JSR LOG
10BA A2 03 LDX #3 COMPUTE NATURAL LOG
*
10C1 0D CD 1D L10 LDA LN10.X
10C4 95 04 STA X2.X LOAD EXP/MANT2 WITH 1/LN(10)
10C6 CA DEX
10C8 10 F0 BPL L10
10CA 20 77 1F JSR FSUB LOG10(X)+LN(X)+LN(10)
10CC 68 RTS
*
10CD 7E 6F * LN10 DCM 0.4342945
10CE 2D ED *
10D0 00 5A R22 DCM 1.4142136 SORT(2)
10D2 92 7A *
10D4 7F 50 LE2 DCM 8.69314718 LOG BASE E OF 2
10D6 05 0C A1 DCM 1.2920074
10D8 00 40 *
10DA 01 A0 MB DCM -2.6390577
10DC 06 49 *
10DE 00 6A C DCM 1.6567626
10E0 00 66 *
10E2 7F 40 MHLF DCM 0.5
10E4 00 00 *
*
10E0 $1000 STARTING LOCATION FOR EXP
*
* EXP OF MANT/EXP1 RESULT IN MANT/EXP1
*
10E0 A2 03 EXP LDX #3 4 BYTE TRANSFER
10E2 00 D0 1E LDA L2E.X
10E4 95 04 STA X2.X LOAD EXP/MANT2 WITH LOG BASE 2 OF E
10E6 CA DEX
10E8 10 F0 BPL EXP+2
10EA 20 77 1F JSR FSUB LOG2(E)*X
10EC A2 03 LDX #3 4 BYTE TRANSFER
10EE 05 00 FSA LDA X1.X
10F0 95 10 STA Z.X STORE EXP/MANT1 IN Z
10F2 CA DEX
10F4 00 FSA BPL FSA SAVE Z+LN(2)*X
10F6 95 10 JSR FIX CONVERT CONTENTS OF EXP/MANT1 TO AN INTEGER
10F8 A5 0A LDA M1+1
10FA 95 1C STA INT SAVE RESULT AS INT
10FC 36 SEC SET CARRY FOR SUBTRACTION
10FE E3 7C SBC #124 INT-124
1100 05 09 LDA M1
1102 E9 00 SBC #0
1104 10 15 BPL OVFLW OVERFLOW INT>124
1106 18 18 CLC CLEAR CARRY FOR ADD
1108 A5 0A LDA M1+1
110A 69 70 ADC #120 ADD 120 TO INT
110C 05 09 LDA M1
110E 20 00 ADC #0
1110 10 00 BPL CONTIN IF RESULT POSITIVE CONTINUE
1112 A9 00 LDA #0 INT<120 SET RESULT TO ZERO AND RETURN
1114 A2 03 LDX #3 4 BYTE MOVE
1116 95 00 STA X1.X SET EXP/MANT1 TO ZERO
1118 CA DEX
111A 10 F0 BPL ZERO
111C 68 RTS RETURN
*
1130 00 * OVFLW BRK OVERFLOW
*
1130 20 2C IF CONTIN JSR FLOAT FLOAT INT
1132 A2 03 LDX #3
1134 05 10 ENT0 LDA Z.X
1136 95 04 STA X2.X LOAD EXP/MANT2 WITH Z
1138 CA DEX
113A 10 F9 BPL ENT0
113C 20 4A IF JSR FSUB Z-Z-FLOAT(INT)
113E A2 03 LDX #3 4 BYTE MOVE
1140 05 00 ZSAV LDA X1.X
1142 95 10 STA Z.X SAVE EXP/MANT1 IN Z
1144 95 04 STA X2.X COPY EXP/MANT1 TO EXP/MANT2
1146 CA DEX
1148 10 F7 BPL ZSAV
114A 20 77 1F JSR FSUB Z#Z
114C A2 03 LDX #3 4 BYTE MOVE
114E 00 DC 1E LA2 LDA A2.X
1150 95 04 STA X2.X LOAD EXP/MANT2 WITH A2
1152 05 00 LDA X1.X
1154 95 10 STA SEXP.X SAVE EXP/MANT1 AS SEXP
1156 CA DEX
1158 10 F4 BPL LA2
115A 20 50 1F JSR FADD Z#Z+A2
115C A2 03 LDX #3 4 BYTE MOVE
115E 00 E0 1E LB2 LDA B2.X
1160 95 04 STA X2.X LOAD EXP/MANT2 WITH B2
1162 CA DEX
1164 10 F0 BPL LB2
1166 20 9D 1F JSR FDIV T=B2/(Z#Z+A2)
1168 A2 03 LDX #3 4 BYTE MOVE
116A 05 00 DLOAD LDA X1.X
116C 95 14 STA T.X SAVE EXP/MANT1 AS T
116E 00 E4 1E LDA C2.X
1170 95 00 STA X1.X LOAD EXP/MANT1 WITH C2
1172 05 10 LDA SEXP.X
1174 95 04 STA X2.X LOAD EXP/MANT2 WITH SEXP
1176 CA DEX
1178 10 F0 BPL DLOAD
117A 20 77 1F JSR FSUB Z#Z+C2
117C 20 1C 1F JSR SWAP MOVE EXP/MANT1 TO EXP/MANT2
117E A2 03 LDX #3 4 BYTE TRANSFER
1180 95 14 LTHP LDA T.X
1182 95 00 STA X1.X LOAD EXP/MANT1 WITH T
1184 CA DEX

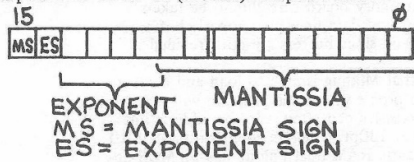
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A 16-BIT FLOATING POINT PROPOSAL

In past weeks, I have talked to several members of the CACHE about "tiny languages." I keep hearing, "I'd use it if it only had floating point." Having written three languages myself, I can understand this. Nobody seems to realize that 32 bits are a lot more than twice as hard to work with as 16.

As a compromise I propose 16 bit floating point. The format I have worked out gives 3 significant digits with an exponent of -15 to +15 (decimal). Proposed format:



I don't have the time or ambition to write this now, but I would be happy to swap ideas with anyone interested.
 Bob Van Valzah (312) 852-0472 (Home)
 1140 Hickory Trl. (312) 971-2010 Ext. 231
 Downers Grove, IL 60519 (Work)

6800 MOTOROLA FOR SIMULTANEOUS NUMBER CRUNCHING AND ANTENNA POINTING

Dear Sir, 17 Nov. 1976

Two of us here in the Northern Virginia area are interested in using a micro for some number crunching (with a peripheral calculator chip) and antenna pointing for satellite work (simultaneously). The 6800 Motorola line of chips looks like it will fill the bill due to the superior I/O configuration possible. The 8080 kinda misses the boat. So I am interested in all kinds of homebrew hardware for 6800 line compatible with SWTP line.

Sincerely,
 Ellis Marshall, W4JK Rt. 1, Box 158
 Front Royal, VA 22630

FREDDIE'S FOLLY

by Jim Day

Frugal Freddie bought a video board kit from a local computer store a couple of months ago. He saved a few bucks by not busying sockets for the ICs. "Who needs 'em?" he said. "I'll just solder everything." The board worked fine for a few weeks, then developed a hardware glitch that Freddie hasn't been able to track down. He took it back to the computer store and asked them what it would cost to fix.

"Well now," said the repairman, "If this thing had sockets, I'd probably find the trouble in a few minutes by random substitution. But with everything soldered down to the board, there's no telling how long it might take. Why, it could end up costing you more than the price of the kit!"

One can avoid duplicating Freddie's folly by socketing everything.

Socket it to 'em, Freddy!

HAMATIC NOTE IN BYTE

According to a letter in the (excellent) November issue of *Byte*, hams who are also interested in computer phreaquery should tune to 3.865 MHz (LSB) on Thursdays at 2300 GMT "for a good time."

ERRATA FOR RANKIN'S 6502 FLOATING POINT ROUTINES

Dear Jim,

Sept. 22, 1976

Subsequent to the publication of "Floating Point Routines for the 6502" (Vol. 1, No. 7) an error which I made in the LOG routine came to light which causes improper results if the argument is less than 1. The following changes will correct the error.

1. After: CONT JSR SWAP (1D07)
 Add: A2 00 LDX=0 LOAD X FOR HIGH BYTE OF EXPONENT
2. After: STA M1+1 (1D12)
 Delete: LDA = 0
 STA M1
 Add: 10 01 BPL *+3 IS EXPONENT NEGATIVE
 CA DEX YES, SET X TO \$FF
 86 09 STX M1 SET UPPER BYTE OF EXPONENT

3. Changes 1 and 2 shift the code by 3 bytes so add 3 to the addresses of the constants LN10 through MHLF wherever they are referenced. For example the address of LN10 changes from 1DCD to 1DD0. Note also that the entry point for LOG10 becomes 1DBF. The routine stays within the page and hence the following routines (EXP etc.) are not affected.

Yours truly,
 Roy Rankin

Dept. of Mech. Eng.
 Stanford University

COMPLETE 8080A FLOATING POINT PKG FOR \$7.50 AND NEW CASSETTE DATA FORMAT STANDARD TO BE PROPOSED

Dear Editor:

Sept. 21, 1976

In response to Paul Holbrook's letter in the September issue, regarding the need for a cassette data format standard, I would like to inform you that a standard with software has been developed; the Mohler standard will be published in an upcoming issue of *Interface*.

The standard allows for various types of data formats and is expandable, so new ones can be added. It is also universal enough for the format to be independent of cassette interface hardware and processor type. We hope to make the Mohler cassette format a standard in the computer hobbyist industry.

I would also like to inform readers that I have developed a single-precision floating point software package for the 8080A (6-7 digits of precision). The package includes add, subtract, multiply, divide, and utility programs to convert from ASCII BCD to binary and binary to packed BCD. It takes up about 1200 bytes and is relatively fast, e.g., 2.5 msec worst case time for multiply.

Also nearing completion is a scientific function package which includes square root, sine, cosine, exponential, natural logarithm, log base ten, arc tangent, hyperbolic sine, and hyperbolic cosine. This package is to be used with the floating point package and takes up less than 1K bytes. It also has six digits of accuracy.

The floating point package is now available for \$7.50. Included are manual, paper tape, and complete annotated source listing. The scientific package will also be \$7.50. Both packages may be ordered for a reduced price of \$10.00. To obtain one or both, send your name, address, and the appropriate amount to:

Burt Hashizume
 P.O. 172
 Placencia, CA 92670